# *The Cosmic Distance Scale III*

## *Measuring the Distances to Galaxies*

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*Themes* : *distance, geometry, gravitation, thermodynamics, photometry, sound-speed.*

*Suggested* : **Measuring Brightness**.

*Suggested* : **Cosmic Distance Ladder I : Measuring the Distance to the Sun**.

**Topics**:



**Resources/Materials**:

- A set of images taken either by the *Hands-On Universe*$^{TM}$ HOU (8 images), or *MACHO* Projects (either field #1 with 13 or field #2 with 25 images)
- A spreadsheet program (e.g. *Excel*)
- A **worksheet** which should be enough for the HOU and MACHO field #1 datasets.
- An *Exel* **spreadsheet** which can be used for an advanced period search (see the section **What to do when you can't see the Period** )

**Activities**:

- **A Theoretical Challenge : What Happens When a Star Pulsates?**

**Links**:

- EU-HOU www.euhou.net
- HOU www.handsonuniverse.org
- MACHO wwwmacho.mcmaster.ca
- OGLE www.astrouw.edu.pl

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# *Introduction*

Distances are some of the most fundamental pieces of information about astronomical bodies in the Universe. Without having the distance, we cannot know the total intrinsic power output of an object - is the object we see in an astronomical image a nearby asteroid only a kilometer across, a star in the Sun's neighborhood, or is it really a Quasar shining with the light equivalent to a trillion suns?. Without knowing the distance, we cannot convert apparent motions on the sky to real space velocities - is the object moving in a series of images a Near-Earth Object (a small asteroid which could wipe us out like it did the dinosaurs) traveling at some tens of kilometers per second, an old star in our Galaxy traveling at hundreds of kilometers per second, or magnetic blobs of gas in a jet being expelled from a distant Quasar at nearly the speed of light? By obtaining distances to faint galaxies, we can learn about how the Universe is evolving and hence what fraction of the Universe is possibly made up of mysterious Dark Matter and Dark Energy.

The astronomical objects visible at various distances are, of course, very different: at great distances, we can only see objects which are intrinsically bright. Within our Solar System, we see the planets, asteroids, and comets. In the solar neighborhood, we can only see normal stars like our Sun, since the planets around neighboring stars are (still) not visible against the bright light of their suns. Far out in our Galaxy, even bright objects like our Sun become very difficult to see: we can only see rather luminous stars hundreds or thousands of times intrinsically brighter than our Sun. In other galaxies, even these very bright stars become difficult or impossible to discern as individual objects and what we see is the uniform light produced by billions of stars. Thus, the methods of measuring the distances are very different when going from the Solar System to the most distant galaxies at the end of the visible Universe. One speaks of the COSMIC DISTANCE LADDER : we need many "*rungs*" (methods) on this ladder to be able to work our way from the Earth to the ends of the Universe, each rung built upon the information gained at a shorter distance.

Concept

DISTANCE LADDER

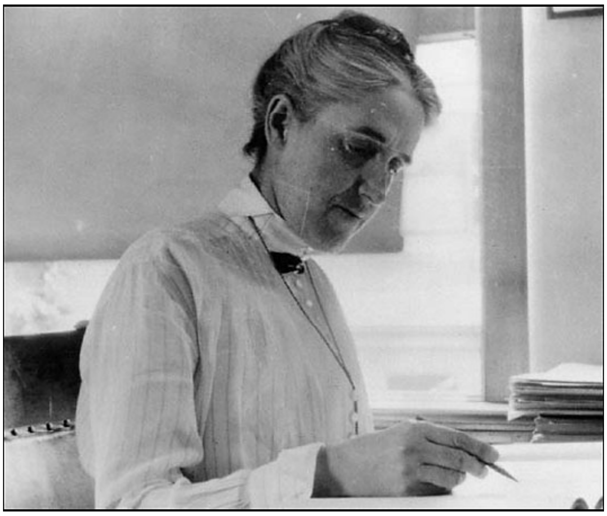
There are dozens of ways to measure cosmic distances, sometimes even at the same distance scale. For instance, we can measure the distance to stars within our own Galaxy using direct geometrical methods, measuring the brightness of well-identifiable stars of known brightness, by following the motions of whole clusters of stars (statistical methods), or by catching them in eclipsing binaries. Such "double-checking" is very important in order to be able to determine the accuracy with which the distances have been calculated, since any errors made at one scale usually propagate to the next larger scale: before we can use stars to measure the distance to external galaxies, we have to know how intrinsically bright such stars are in our own Galaxy. Nevertheless, advances in analysis techniques, our theoretical understanding of how stars work, and the opportunities for very precise observations provided by large groundbased and space telescopes are reducing the number of important "rungs" in the modern "Distance Ladder" and thus providing us with more accurate distances.

Direct (i.e. geometric) distances to astronomical objects â€“ e.g. parallaxes to neighboring stars â€“ are usually only possible within a fairly small neighborhood around the sun. While indirect methods are available to determine the distances to normal stars in our galaxy, most of them fail when applied to external galaxies because of the tremendous distances involved â€“ it is impossible to see individual "dwarf" stars like our sun because they are simply too faint. Red giant and supergiant stars are much more luminous and can be seen individually in at least nearby galaxies, but they have a wide range of nearly random luminosities. What is needed is a cosmic yardstick â€“ a STANDARD CANDLE â€“ whose LUMINOSITY (intrinsic power output) is simply correlated with readily observable properties.

Concept

STANDARD CANDLE
LUMINOSITY

The means for bridging the distance gap between our Galaxy and external galaxies was discovered by Henrietta Swan Leavitt in 1912. Ms. Leavitt was born in Cambridge, Massachusetts in 1868, the daughter of a Congregational minister. While studying at Oberlin and Radcliffe Colleges, she became interested in astronomy and, after graduation, worked voluntarily at Harvard College Observatory as a "computer" (someone who performed tedious calculations on a mechanical calculator). In 1902, she obtained a staff position (at 30 cents/hour) and eventually rose to become the head of the photometry group â€“ a team of women whose job it was to estimate the brightness of thousands of stars recorded on photographic plates. Her boss, Edward C. Pickering, assigned her the duty of searching for variables in hundreds of photographs taken of the Small Magellanic Cloud (SMC), a small irregular galaxy orbiting around our own Galaxy. This job involved the tedious comparison of a series of images of the same field using a "blinking" technique to find which stars change their brightness relative to the majority which remain constant.



Henrietta Swan Leavitt (1868-1921)

Leavitt ultimately found 1777 variables in the SMC and would, in principle, have been finished with her work. However, as a true astronomer, she was interested in the reason why the stars were variable, and found that the period of the variations in a small subset of the stars correlated with the brightness: the brighter stars have longer periods. Leavitt, being a female assistant and not a "real astronomer", was not allowed to publish her findings directly, so Pickering published her findings in the Harvard College Observatory Circulars (No 173, March 3, 1912) for her:

"*The following statement regarding the periods of 25 variables in the Small Magellanic Cloud has been prepared by Miss Leavitt. ... A straight line can be readily drawn among each of the two series of points corresponding to maxima and minima,* [see Fig. 1 above] *thus showing that there is a simple relation between the brightness of the variable and their periods. ... Since the variables are probably the same distance from the Earth, their periods may be associated with their actual emission of light, as determined by their mass, density, and surface brightness.*"

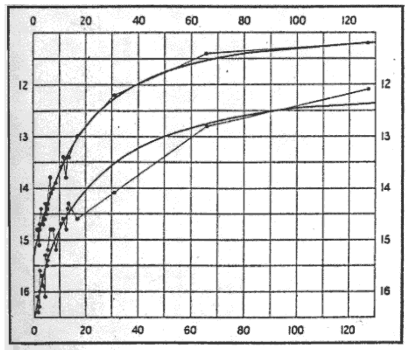


Fig. 1.

Leavitt's original Period-Brightness relation (X-axis in days, Y-axis in magnitudes).

The size of the SMC is indeed much less than its distance from us, so Leavittâ€™s correlation implies a connection between the periods and the intrinsic brightness. Leavitt died of cancer in 1921 and was nominated for the Nobel Prize in 1925 for her pioneering work, particularly the recognition of the PERIOD-LUMINOSITY RELATION but also for her efforts to define a precise photometric brightness scale for stars.

Concept

PERIOD-LUMINOSITY RELATION

Soon after Leavittâ€™s discovery, the Danish astronomer Ejnar Herzsprung realized that, once calibrated, the PL relation could be used to estimate the intrinsic brightness of a star solely on the basis of its pulsational period (something which Leavitt had implied but not clearly stated in her publication). Due to the resemblance of the SMC variables to a known variable star in our Galaxy, Î´ Cephei (the fourth brightest star in the constellation Cepheus, known to be variable since 1784), Herzsprung labeled them "Cepheids". With the help of Cepheids within our Galaxy, Herzsprung obtained a rough calibration and published a distance of 3,000 parsecs (1 parsec = 3.26 light years = $3.09 \times 10^{13}$ km) to the SMC (probably a typo â€“ Herzsprungâ€™s own numbers imply a distance of 30,000 parsecs, still a factor of about 5 too small of the modern value but a gigantic distance for those days). Harlow Shapely revised Herzsprungâ€™s calibration and used it to derive an accurate size of our Galaxy, whose large value caused considerable debate at the time: the PL relation played a part in the "Great Debateâ€□ between Shapely and Curtis in 1920 on whether external galaxies were systems like our own Milky Way. With the discovery of Cepheids in the Andromeda Galaxy by Edwin Hubble in 1926 and the determination of the huge distances to external galaxies like our own, it finally became clear that the Milky Way is only the nearest of these â€œIsland Universesâ€□ and that the universe is unimaginably large.

Cepheids continue to be a major field of investigation in modern astrophysics simply because they remain the best understood and most practicable "Standard Candles" which can be calibrated in our Galaxy and applied to fairly distant external galaxies. We now know that the PL relation is altered by the amount of heavy elements within the stars (really old stars have less and really young stars have more than the sun) and that there are Cepheids with more complicated pulsation patters (e.g. Polaris, the North Star, is such an â€œovertone-pulsatorâ€□). One of the major scientific programs ("Key Projects") for the Hubble Space Telescope was to observe Cepheid variables in distant galaxies, from which one of the best estimates for the global expansion of the universe has been derived: the techniques used for the Hubble data are qualitatively the same as those which we will used in this activity.

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## *The Physics of Pulsating Stars*

Cepheids are pulsating giant stars which periodically change their radii, temperature, and brightness. Practically all stars pulsate somewhat: for example, convective motions in the upper layers of the sun create large-scale waves with characteristic periods of about 5 minutes. These wave motions penetrate deep into the interior of the sun and can be used to probe the sunâ€™s internal structure (e.g. temperature, density, and rotation). Cepheids, on the other hand, show much more dramatic and global variations: the entire star periodically grows and shrinks back again radially on timescales of days to months.

The radial pulsational timescale can be estimated by considering the time it takes a sound wave to pass through the star â€“ the latter is the speed at which pressure information can propagate â€“ since the different layers of the star must be able to communicate with each other if the stars to pulsate as a whole. If the pulsations are not too strong, the star is always very close to hydrostatic equilibrium â€“ the force of gravity at a radius r is balanced by local pressure forces created by a radial gradient dP/dr in the thermal pressure P â€“ and then

$$
\begin{aligned}
dP(r)/dr &= -g(r)\ \rho(r) \\
&\approx -\Delta P / \Delta r \\
&\approx -\langle P \rangle / R \\
&\approx -g(R)\ \langle \text{Ï□} \rangle
\end{aligned}
$$

where R is the radius of the star, $g(r) = G\ M(r) / r^2$ is the acceleration due to gravity (e.g. in meters $s^{-2}$), and $\langle P \rangle$ and $\langle \rho \rangle$ are the mean pressure and mass-density (kg $m^{-3}$). The (square of the) speed of sound cs is then

$$c_s^2 \approx \langle P \rangle / \langle \rho \rangle \approx R\ g(R) \approx G\ M / R$$

and the resulting time Π for a sound wave to propagate from the center to the surface of the star is roughly

$$
\begin{aligned}
\Pi &\approx R / c_s \\
&\approx (R^3 / G\ M)^{1/2}
\end{aligned}
$$

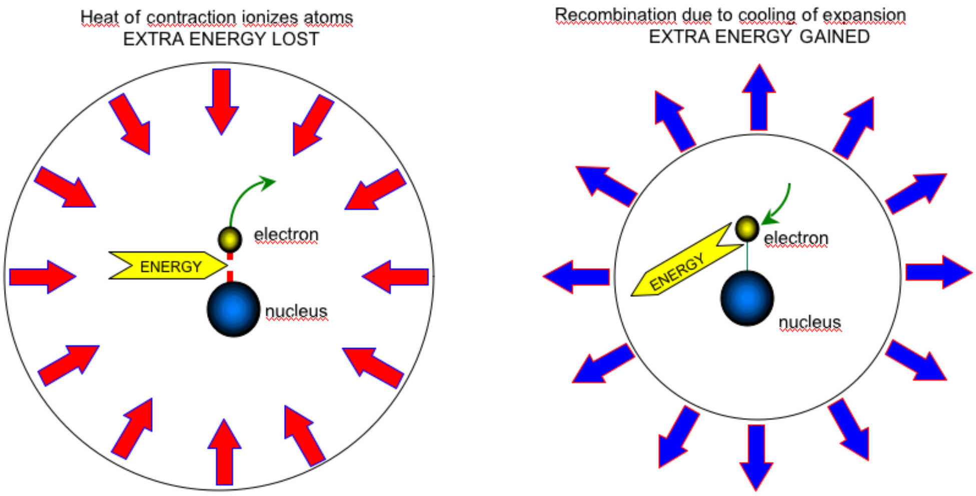
$$\approx (4 \pi G \langle \rho \rangle / 3)^{-1/2}$$

$$\approx 27 \text{ minutes } (M / M_{Sun})^{-1/2} (R / R_{Sun})^{3/2}$$

The actual value depends upon the details of the stellar structure, the thermodynamics of the dense stellar material, and the amount of heavy elements present (they largely determine how opaque the star is to the radiation coming out of its interior). Note that the non-radial 5-minute wave-motion on the sunâ€™s surface is faster than the much deeper radial pulsations would be. Cepheids typically have masses of 4-14 $M_{Sun}$ and radii of 14-200 $R_{Sun}$ and so have pulsational periods from many days to weeks and months.

The equation for the pulsation period looks a lot like that describing the tone of a bell: the larger the size (R), the deeper the tone (lower frequency = larger period) and the more dense the material (<Ï□>), the higher the tone.

As the star shrinks, the interior of the star becomes hotter, and as the star expands, the star cools. Unless the pulsation is driven or is strictly adiabatic (no exchange or transformation of the internal heat energy), the intrinsic viscosity of any material will cause the ringing of the star (or a bell) to die out eventually.




Schematic diagram showing the radial pulsation of a star and how ionization and recombination during expansion and contraction effects the flow of heat.

The source of the pulsations in most variable stars is an intrinsic instability caused by the exchange of thermal and chemical energy within the envelope of the star: as the star contracts (figure above), the increase in temperature can ionize matter (e.g. hydrogen), which removes thermal energy and hence pressure from the gas (it takes energy to remove the electrons from the nuclei) and causes the star to contract even more. When the rebound occurs and the star starts to cool again, the ions and electrons re-combine, releasing the energy of the formerly free electrons: this additional heat causes the star to expand even more before the whole cycle starts again. Mathematically, one can show that, the star will happily and continuously pulsate, taking the energy it needs from the radiation streaming out from the starâ€™s interior, if there is the right time-shift between the normal pulsations and this release and trapping of chemical energy. This is analogous to the operation of an automobile engine: the engine runs smoothly only if the fuel burns when the expanding gases can push the pistons and so drive rather than brake the engine.

Because the pulsations are nearly adiabatic, the actual bolometric luminosity of the star (the total amount of radiation given off at all wavelengths per unit time, in Watts) doesnâ€™t change appreciably during a single pulsational cycle: the visible light changes dramatically, however, because both the surface temperature and the surface area vary considerably.

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## *The Measured Quantities*

A star radiates energy in the form of light in all directions. This light is a stream of photons whose density (photons per unit volume) decreases with increasing distance. We measure the stream of photons at some distance from the star with a detector (generally a digital camera and a colored filter) of some given area over some period of time. Thus, we must distinguish between the amount of energy radiated by the star â€“ an intrinsic quantity independent of the observer â€“ and the amount of energy we observe â€“ a quantity dependent upon the observerâ€™s distance and apparatus.

**The LUMINOSITY L is the total amount of energy radiated by the star in all directions per unit time (e.g. in Watts).**

Concept

LUMINOSITY

For our purposes, it will be handy to define the luminosity L(band) as the total amount of energy radiated in some wavelength band (e.g. in â€œredâ€□ light, $L_R$) in all directions per unit time interval (also measurable in Watts). The wavelength band is set by the type of filter and spectral sensitivity of the detector used to make the observation. Obviously, $L_R$ (for instance) can only be a fraction of the total (bolometric) luminosity L, and even if L is constant (as in Cepheids), changes in temperature (color) and size can shift parts of the luminosity between different bands (e.g. between the red and blue parts of the optical spectrum).

What we can directly observe is not the intrinsic luminosity but the APPARENT BRIGHTNESS given that we are some distance d away from the star and have an instrument which can collect a fraction of the light from the star within some time period. The physical quantity for apparent brightness is the flux F:

**The apparent brightness or FLUX F is the amount of energy per time interval and area seen by an observer some Distance d from the star (e.g. in Watts/$m^2$).**

Concept

APPARENT BRIGHTNESS
FLUX

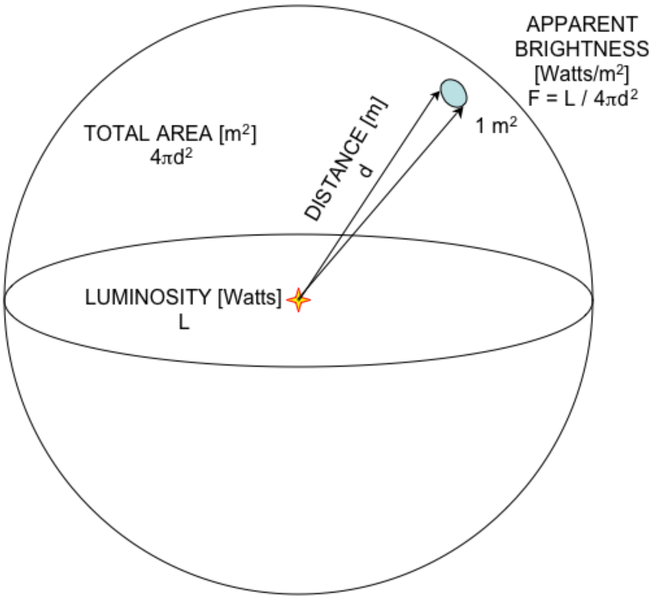
Similarly, it will be handy to define the flux F(band) as the flux observed only within some wavelength band (e.g. in â€œredâ€□ light, $F_R$)(also measurable in Watts/$m^2$). Astronomer traditionally use a totally different means of measuring flux: the starâ€™s brightness in â€œmagnitudesâ€□ relative to some standard stars, a unitless inverse logarithmic scale originally invented by the Greek astronomer Ptolemy and which is often handy to use, but which we will ignore for now.

The three quantities L, F and d (or $L_R$, $F_R$ and d) are intimately connected: the observed flux is simply the luminosity spread out over the surface of a sphere with radius d and surface area 4 π $d^2$:

$$F = L / 4\pi d^2$$

Thus, in order to find the distance d to a star, we need to know itâ€™s intrinsic luminosity L and the observed flux F. Since the flux from a variable star varies â€“ by definition â€“ we will use the mean flux averaged over the pulsational cycle:

$$\langle F_R \rangle = \langle L_R \rangle / 4\pi d^2.$$




# *The Method*

In the previous section, we learned that the Luminosity is the total amount of power produced by a star, in our case, in a particular wavelength region defined by a filter. What we can measure is just the apparrent brightness or flux F which is produced by collecting a small fraction of the total luminosity of a star at a large distance d. If we know L and know F, we can use the relation $F = L / 4\pi d^2$ and solve for d.

In order to find the distance to a Cepheid variable star, we can obtain the star's Luminosity (e.g. in Watts) using the known relationship between the pulsation period and the intrinsic luminosity, and the star's apparent brightness (e.g. in Watts/$m^2$). The necessary steps are then:

1. **Determine the pulsational period P by analyzing the brightness changes of the Cepheid relative to a constant star as a function of time in a series of astronomical images taken through some filter.**
2. **Determine the intrinsic luminosity in the filter's wavelength range, $L_{Filter}$, using an appropriate Period-Luminosity relation for that filter.**

3. **Find the mean apparent brightness $<F_{Filter}>$ by measuring the mean brightness of the Cepheid star (averaged over the pulsational cycle) relative to a standard star of known apparent brightness.**
4. **Calculate the distance d from the measured mean flux $<F_{Filter}<$ and $L_{Filter}$ using the equations above.**

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## *The Image Data*

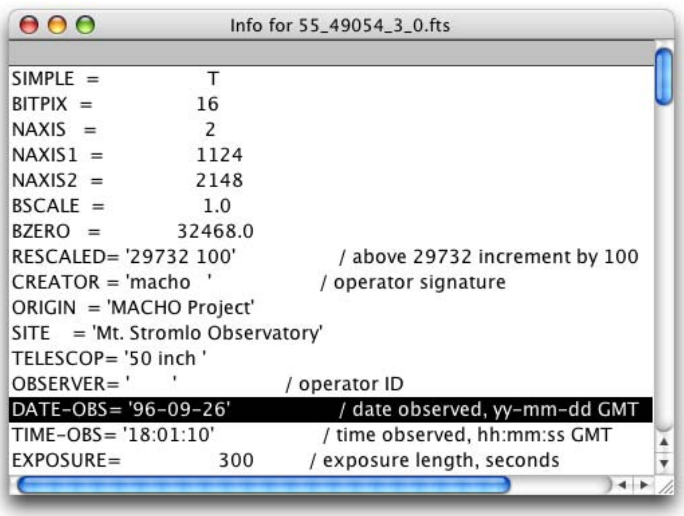
Several image data sets are available which can be used for this exercise:

- The *Hands-On Universe™* (HOU) dataset consists of 8 V-filter images of a Cepheid variable in our galaxy and a nearby star which serves as the REFERENCE STAR of *known* apparent brightness. This dataset is ideal for training purposes or when there isn't enough time for a larger dataset.
- The datasets prepared for use by Göttingen's *XLAB* consist of two fields within the *Large Magellanic Cloud* taken from the database of the *MACHO Project* : the first one ("field #1") contains 5 faint Cepheids and a reference star; the second one contains 3 Cepheids of different brightnesses and a reference star. Both datasets contain images made in a special MACHO red filter for which a MACHO Period - Red-Luminosity diagram has been prepared.
- The *EU-HOU* (European Hands-On Universe) *Project* has prepared a similar dataset of 20 Large Magellanic Cloud images taken from the *Optical Gravitational Lensing Experiment* (OGLE) database.

Concept

REFERENCE STAR

The files are stored in the so-called â€œFITSâ€œ format which permits detailed information about the image to be stored along with the actual image data (e.g. where the observation was made, who made it with which instrument). This "Header" information can be easily listed: select the menu entry `Image > Show Info...` and look for the keywords DATE-OBS and TIME-OBS. Here, we are not interested in the


Info for 55_49054_3_0.fts

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SIMPLE  =                T
BITPIX  =               16
NAXIS   =                2
NAXIS1  =             1124
NAXIS2  =             2148
BSCALE  =              1.0
BZERO   =          32468.0
RESCALED= '29732 100'          / above 29732 increment by 100
CREATOR = 'macho   '           / operator signature
ORIGIN  = 'MACHO Project'
SITE    = 'Mt. Stromlo Observatory'
TELESCOP= '50 inch '
OBSERVER= '        '           / operator ID
DATE-OBS= '96-09-26'           / date observed, yy-mm-dd GMT
TIME-OBS= '18:01:10'           / time observed, hh:mm:ss GMT
EXPOSURE=              300     / exposure length, seconds
```

absolute time of the observations but the relative time measured with respect to some convenient moment (e.g. days since the first observation). If these (or similar) keywords are present you can choose to let `ImageJ` calculate the exact time of the observation (the *Julian Date*).

Usually, there are lots of stars in the images, so finding the right ones isnâ€™t always easy. For that purpose, we have need a "finding chart" â€“ an image where the stars of interest are indicated. By recording and comparing the positions of stars (indicated at the bottom right hand corner of the windows) and noticing random patterns of stars, you should be able to find your way around. The HOU dataset doesn't need any and the MACHO and OGLE datasets include finding charts.

The FITS header information obtained via the menu command `Image > Show Info....`

**Table I : image datasets**

| Link to Dataset | Number of Images | Filter | Apparent brightness $F_{filter}$ of Reference Star | Description |
|---|---|---|---|---|
| *Hands-On Universe™* | 8 | V (standard visual) | $2.28 \times 10^{-12}$ W/m$^2$ | simple galactic Cepheid |
| *MACHO* LMC field #1 | 13 | red (non-standard) | $2.19 \times 10^{-15}$ W/m$^2$ | very faint Cepheids, so periods not obivous at first |
| *MACHO* LMC field #2 | 25 | red (non-standard) | $2.33 \times 10^{-15}$ W/m$^2$ | Cepheids of different brightnesses |

The Large Magellanic Cloud (LMC) is an irregular (as opposed to a spiral or elliptical) galaxy orbiting the Milky Way. The galaxy contains lots of dust and gas clouds where new stars are being born (see the cover image). Many of the more massive new stars have already finished their hydrogen-burning phase and have inflated themselves into hot giant stars susceptible to the Cepheid pulsational instability. Thus, there are thousands of Cepheid variables in the LMC which have only relatively recently been found by intensive photometric campaigns.

We will assume you're using the data taken on a 1.4m telescope obtained by the so-called â€œMACHO Projectâ€□ at Mt. Stromlo in Australia. The MACHO Project didnâ€™t set out intending to discover Cepheids, but rather rare brightenings of random LMC stars due to the gravitational lensing effects of â€œMassive Compact Halo Objects â€“ faint objects which might make up most of the mass in the halo of our galaxy â€“ positioned about halfway between us and the LMC. As it turns out, they didnâ€™t find very many MACHO candidates but they did find a large number of other normal variable stars, including thousands of Cepheids. Afterwards, the MACHO consortium generously have opened access to their data over the internet, so we can use it for our own purposes. Tragically, the telescope was recently destroyed by forest fires!

Due to the amount of dust in the LMC, itâ€™s dangerous to look for Cepheids in blue or green light: this light is easily absorbed or scattered by dust grains, making the stars look fainter and the distances larger than they really are. Thus, the red-band MACHO data were extracted for this exercise.

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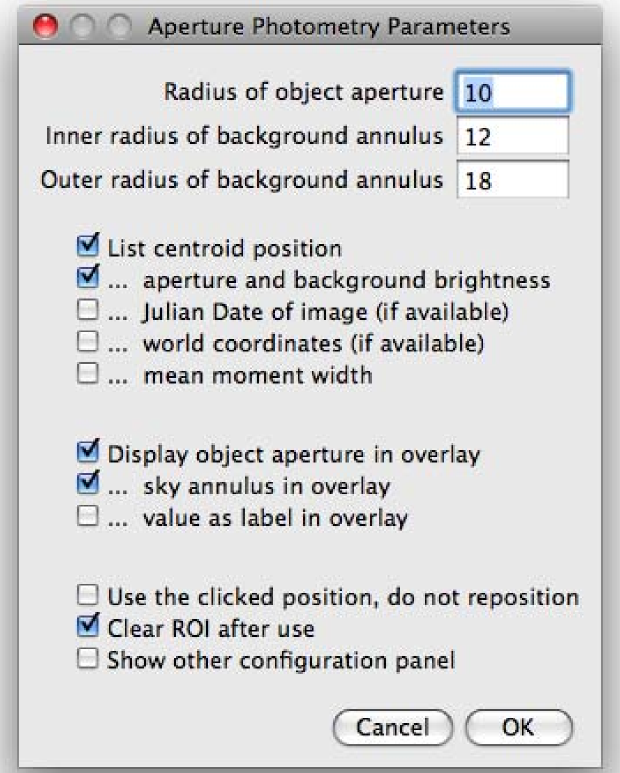
## *Measuring the Pulsation Period*

We need to measure the brightness of the Cepheid and reference stars in the images in order to see the relative light variations of the Cepheid. The light from stars in astronomical images is spread out over a small range of pixel due to the finite optical resolution of the telescope and due to smearing of the image caused by turbulence in the Earthâ€™s atmosphere. If we want to measure all the light from the star, we need to add up the light in an aperture centered on the star. At the same time, we need to subtract the background light present in each pixel. This adding up of signal over a circular area in the images is called "aperture photometry".

For more information about aperture photometry - particularly the choice of the aperture radii - see the chapter ▸**Measuring Brightness**◂. Fortunately, we are only interested in the relative brightness of two stars in the same image, so the choice of aperture radii is not so important. Note, however, that crowded fields - images containing lots of stars all packed closely together - are tricky.

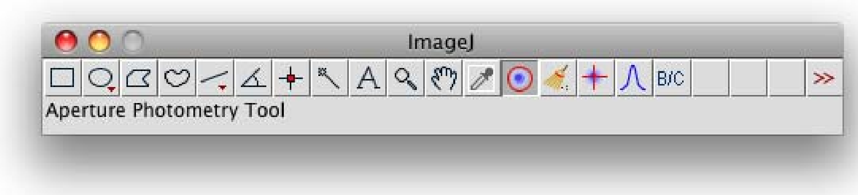


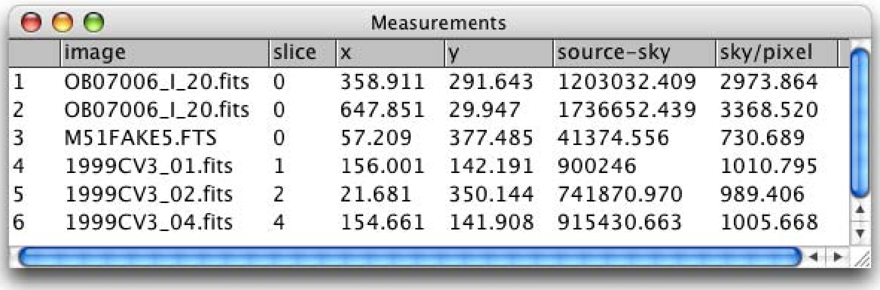

Check to make sure that the right measurements will be displayed : invoke `Plugins > Astronomy > Set Aperture` and make sure you have selected at least the options "Display centroid position", "Display aperture and background brightness" ("Display Julian Date of image (if available)" will result in the output of a convenient measure of relative time - if the images have the right info).

Once the aperture radii and the measurement values to be displayed have been chosen, the measurement is very simple: click on the *Aperture Photometry Tool* icon in the `ImageJ` toolbar and then again on the star to be measured. The result is shown in a *Measurements* window: e.g.

- measurement number;
- image name (and stack position, if measured within an image stack);
- X- and Y-position of the star (in pixels);
- total integrated brightness of the star minus the corresponding sky background;
- mean value of the sky background (per pixel);
- Julian Date (the number of days which have elapsed since Monday, January 1, 4713 B.C.E.)

The units of the brightness and sky values are uncalibrated raw "counts" C.





Measurements

| | image | slice | x | y | source-sky | sky/pixel |
|---|---|---|---|---|---|---|
| 1 | OB07006_I_20.fits | 0 | 358.911 | 291.643 | 1203032.409 | 2973.864 |
| 2 | OB07006_I_20.fits | 0 | 647.851 | 29.947 | 1736652.439 | 3368.520 |
| 3 | M51FAKE5.FTS | 0 | 57.209 | 377.485 | 41374.556 | 730.689 |
| 4 | 1999CV3_01.fits | 1 | 156.001 | 142.191 | 900246 | 1010.795 |
| 5 | 1999CV3_02.fits | 2 | 21.681 | 350.144 | 741870.970 | 989.406 |
| 6 | 1999CV3_04.fits | 4 | 154.661 | 141.908 | 915430.663 | 1005.668 |

When you have measured the brightness (in "counts") of your Cepheid star - call it "$C_{Cepheid}$" - and that of the reference star - call it "$C_{ref}$", you can form their ratio $R = C_{Cepheid}/C_{ref}$ representing the *relative* Brightness of the star with respect to the standard star. Since the ratio of the brightnesses in the image is the same as the apparent brightness or flux, we can calculate the apparent brightness of the Cepheid by solving $R = F_{Cepheid}/F_{ref}$ for $F_{Cepheid}$.

Find one of the Cepheids marked in the finding chart. Measure the brightness of the Cepheid and the marked standard stars (labeled with something like "R=14.70" in the MACHO data) over the time period covered by the set of images, placing the results in your worksheet or in a spreadsheet. The dates and times are recorded in the FITS image headers: if you didn't opt to have the aperture measurement automatically print out the (Modified) Julian Date, note BOTH the date and time (the latter is usually called â€œUTâ€□ for â€œUniversal Timeâ€□ or â€œTIME-OBSâ€□ for â€œtime of observationâ€□) as you may need a high time accuracy if the period is short.

When you have measured the relative brightness R of your Cepheid star in each image, you can plot the relative brightness R as a function of the relative time t: this is a LIGHTCURVE and represents the temporal behavior of the starâ€™s relative brightness R(t) in the spectral region defined by the filter and detector.

Concept

LIGHTCURVE

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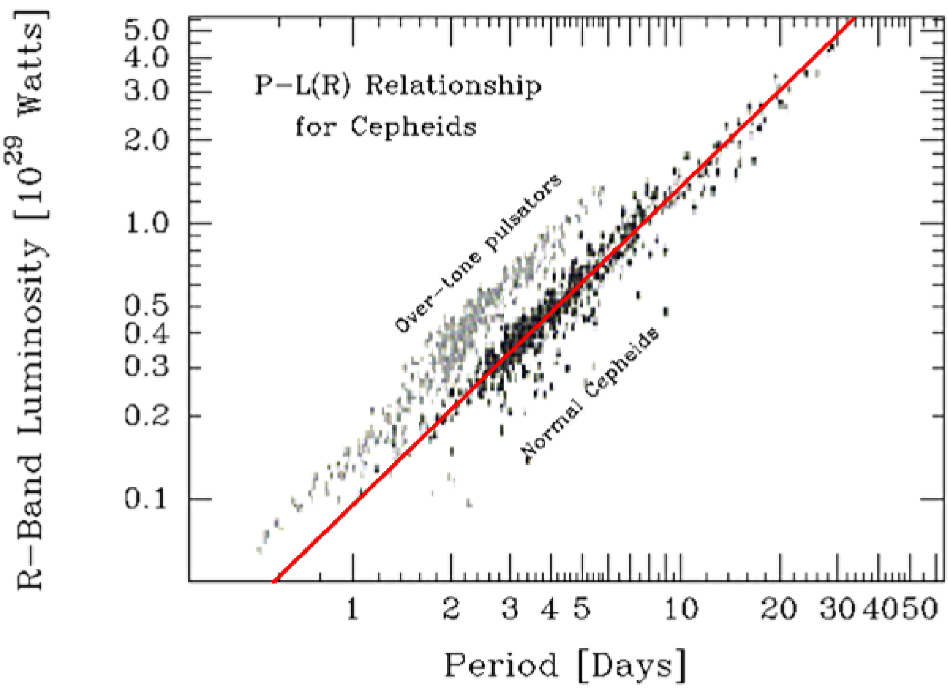
## *Deriving the Luminosity*

Once the pulsational period has been found from inspection or analysis of the lightcurve, the luminosity can be derived using an appropriate form of the Period-Luminosity relation. "Appropriate" means that one need to use the relation corresponding to the filter used. The HOU observations were made using a standard "V" (or "visual") filter. The MACHO observations were made in a non-standard "red" filter.




The Cepheid Period-Luminosity Relation for the MACHO red filter:
given a pulsation period in days, the red line shows what the star's red-luminosity is in Watts.

---

## *Measuring the Mean Brightness*

Given the intrinsic luminosity or power coming from the Cepheid (in units of Watts, for example), we need the apparent brightness of the star (in units of Watts per $m^2$, for example). The aperture photometry measurements added up the apparent brightness of the star in the uncalibrated "counts" of the images, but we need physical units. One can construct a calibration so that we know how many Watts/$m^2$ correspond to one "count", but there is a much simpler method: the relative brightness light curve of the Cepheid versus the reference star already tells us all we need to know.

**If the mean light level of the Cepheid is *N*% of the reference star, and the reference star has a know apparent brightness $F_{filter}$, then the apparent brightness of the Cepheid is simply 100**N**$F_{filter}$.**

The ratio of the raw counts (Cepheid star divided by standard) is equal to the ratio of the fluxes and is (nearly) independent of all the effects which might have reduced the number of counts (clouds, altitude of the observations, changes in the detector,...).

The "mean light level" needed above is easily obtained by averaging all of the light curve points (assuming that the measurements were taken at random phases, a simple average should be good enough).

## Calculating the Distance

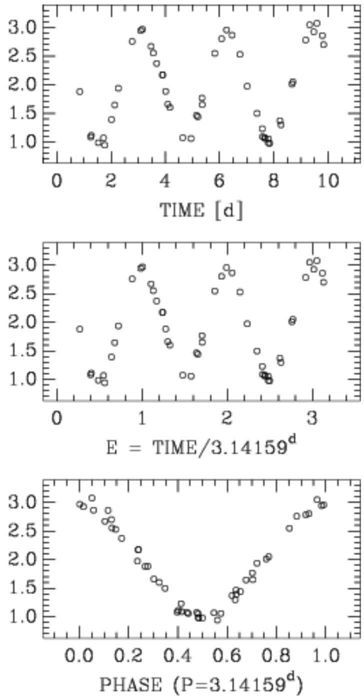
Now that all of the steps have been discussed, here is a summary covering everything which is needed:

1. **Measure the brightness of the Cepheid and the reference star** using the aperture tool in the `ImageJ` toolbar.
2. **Note the dates and times of the observations** (either from the names of the files, from the FIT header information, or already expressed as a Julian Date by the aperture plugin).
3. **Enter the image names, dates & times, Cepheid and reference star brightnesses in the table** below (or in a spreadsheet) for all images in a dataset.
4. **Calculate the the relative brightness** of the Cepheid with time (ratio of the Cepheid and reference star aperture measurements).
5. **Determine the pulsational period** by plotting your data in a lightcurve (relative brightness of the Cepheid versus time).
6. **Using the correct Period-Luminosity relation, determine the luminosity** of the Cepheid in the filter used in Watts.
7. **Compute the average relative brightness** of the Cepheid relative to the reference star.
8. Given the known apparent brightness of the reference star, **calculate the mean apparent brightness of the Cepheid in Watts/m$^2$**.
9. Given the luminosity $L_{filter}$ and mean apparent brightness $\langle F \rangle_{filter}$ of your Cepheid and the equation connecting these two quantities with the distance d, **derive the distance in meters, parsecs, and light-years**.
10. OPTIONAL: Measure the other Cepheids and find the mean distance and the standard deviation. How does your value compare with that in the literature (via Google)? Produce your own Period-Brightness relation. How well does the slope of your relation (log Period versus log Brightness) match the official one? Compare your results with those of your colleagues: if there are any significant differences, can you identify what they might be due to?

These steps are easy if you follow the ▸worksheet◂ or use the ▸spreadsheet◂ .

## What to do when you can't see the Pulsation Period

If youâ€™re lucky, the pulsational period will be obvious when looking at the lightcurve (e.g. the period Î  would be the time between subsequent troughs or peaks). Normally, however, the â€œsamplingâ€□ of the lightcurve is less than ideal â€“ data may be missing for days or weeks in a string of observations due to weather or some other problem â€“ and the lightcurve looks like a random â€œscatter diagramâ€□ For example, the lightcurve to the right represents random samples of an imaginary star with a strictly sinusoidal flux variation (top).




Example of how to "fold" a time-series of observations in phase: (top) original data with a period $P = \pi$; (middle) the same data plotted as a function of number of cycles E; (bottom) the data plotted in phase.

Of course, if we knew the real period, we could plot the data not as a function of time but as the number of cycles which have passed or as the cycle phase (usually measured from 0.0 to 1.0 or sometimes from 0 to $2\pi$): the middle and bottom plots to the right shows exactly the same points but plotted not in order of time but cycles and cycle phase. The latter can be calculated from the time and the period:

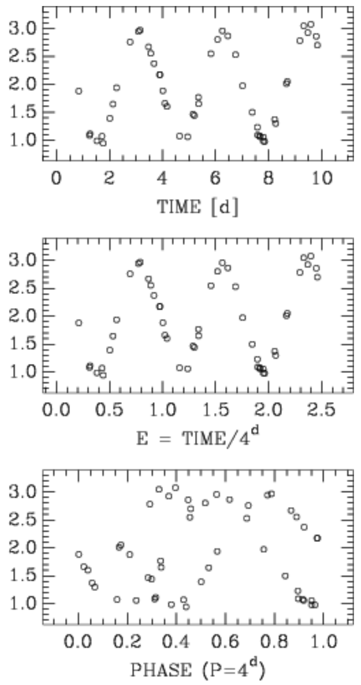
**The Phase Ï† is the fractional part**
**of the number of cycles passed, E :**

$$E \equiv t / P$$

$$\varphi \equiv \mathrm{modula}(E,1) = E - \mathrm{integer}(E)$$

(plus some constant if a particular phase is associated with a particular time)

The quantity E measures the number of decimal cycles elapsed since t=0. If we think about measuring calendar time in either weeks or days, then the phase is something like the "day-of-the-week" for t measured in days and P=7 days. Note that, when the data is plotted using E instead of t, thereâ€™s really no difference. The really obvious signal appears only when the data is â€œfoldedâ€□ together in phase (bottom plot).




Same as the figure above, but using the wrong period: note how the phased data no longer fits together like beads on a string.

This different representation of the time is trivial to compute in a spreadsheet: select some handy spreadsheet field which holds our value for the period P (e.g. `C22` in the figure below): if the relative times t are in the column `F` starting in the row #7, for example, then the normalized time E can be computed in cell `G6` using the function

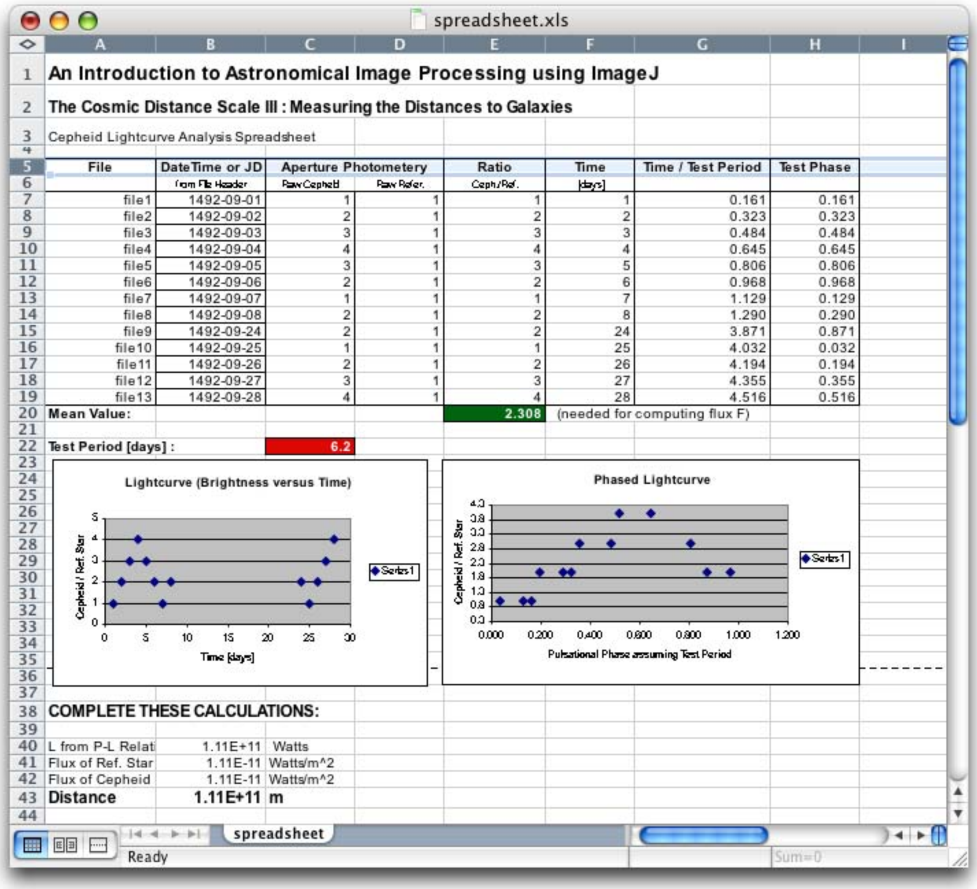
```
=(F7/$C$22)
```

(the dollar-signs mean that we don't want the spreadsheet to change where to find our period, no matter what data point we use later) and the phase φ using the function

```
=G7-INTEGER(G7)
```

(available in German as `=G7-GANZZAHL(G7)`). The values of φ obtained this way should have values between 0 and 1.

Of course, if we donâ€™t know the actual period P, then the phase may be off considerably (imagine calculating the day of the week using weeks of only 6 instead of 7 days so that all the Sundays arenâ€™t lined up but sometimes are put together with Tuesdays and Fridays). However, we do know that a bad period guess will result in a random plot like that to the left, a good period guess should result in the lightcurve producing a closely beaded string of phased points (see figure above), and a nearly OK period will produce something in-between.




| File | DateTime or JD | Aperture Photometery | | Ratio | Time | Time / Test Period | Test Phase |
|---|---|---|---|---|---|---|---|
| | From File Header | Raw Cepheid | Raw Refer. | Ceph/Ref. | [days] | | |
| file1 | 1492-09-01 | 1 | 1 | 1 | 1 | 0.161 | 0.161 |
| file2 | 1492-09-02 | 2 | 1 | 2 | 2 | 0.323 | 0.323 |
| file3 | 1492-09-03 | 3 | 1 | 3 | 3 | 0.484 | 0.484 |
| file4 | 1492-09-04 | 4 | 1 | 4 | 4 | 0.645 | 0.645 |
| file5 | 1492-09-05 | 3 | 1 | 3 | 5 | 0.806 | 0.806 |
| file6 | 1492-09-06 | 2 | 1 | 2 | 6 | 0.968 | 0.968 |
| file7 | 1492-09-07 | 1 | 1 | 1 | 7 | 1.129 | 0.129 |
| file8 | 1492-09-08 | 2 | 1 | 2 | 8 | 1.290 | 0.290 |
| file9 | 1492-09-24 | 2 | 1 | 2 | 24 | 3.871 | 0.871 |
| file10 | 1492-09-25 | 1 | 1 | 1 | 25 | 4.032 | 0.032 |
| file11 | 1492-09-26 | 2 | 1 | 2 | 26 | 4.194 | 0.194 |
| file12 | 1492-09-27 | 3 | 1 | 3 | 27 | 4.355 | 0.355 |
| file13 | 1492-09-28 | 4 | 1 | 4 | 28 | 4.516 | 0.516 |
| Mean Value: | | | | 2.308 | (needed for computing flux F) | | |



The measure of â€œgoodnessâ€œ in our phased lightcurve is thus a minimum of vertical scatter for points at similar test phases. This classic method of finding a non-sinusoidal signal in a random string of observations has a formal name: "Phase-Variance Minimization" and is widely used in all kinds of scientific and technical applications.

This analysis is very simple to carry out using your spread-sheet plot: if the column of numbers representing your test phase computed using your test period â€“ which has handily been placed in a separate spread-sheet cell (e.g. `C22` in the example and figure above) â€“ is plotted, then changing the value of the test period immediately gets plotted as a changed phased data plot, enabling you to search for a good test period by eye.

If you construct your own spreadsheet, make sure to record all the important data and results. Obvious entries include such things as:

- your name, a title and other info as warranted at the very top;
- a row of labels for all the columns below;
- a column of file names;
- a column of dates and times of observation;
- a column of relative time t of observation in decimal days since the first observation;
- a column of raw counts from the Cepheid C;
- a column of raw counts from the Standard Star $C_{ref}$;
- a column of relative brightness of the Cepheid R;

The mean relatvie brightness of the Cepheoid can be derived using the spreadsheet function

`AVERAGE` (for the German version of Excel, use `MITTELWERT`) : if you want to average the values in a particular column range, say A5 to A10, and place the result in a cell, then define the cell using the syntax

```
= AVERAGE(A5:A10)
```

or

```
= MITTELWERT(A5:A10)
```

A complete spreadsheet, waiting to be filled out, is available ▸here◂.

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